

By using conventional definitions of $F_0=0$, $F_1=1$, $F_2=1$ and $F_n=F_{n-2}+F_{n-1}$:

$$\begin{aligned}
 & \frac{F_0}{1} + \frac{3}{5}F_1 + \left(\frac{3}{5}\right)^2 F_2 + \dots + \left(\frac{3}{5}\right)^n F_n + \dots \\
 &= \sum_{n=0}^{\infty} \left(\frac{3}{5}\right)^n F_n \\
 &= \sum_{n=0}^{\infty} \left(\frac{3}{5}\right)^n \left[\frac{\varphi^n - (1-\varphi^n)}{\sqrt{5}}\right] \quad (\text{by substituting the closed form expression for Fibonacci numbers}) \\
 &= \frac{1}{\sqrt{5}} \left[\sum_{n=1}^{\infty} \left(\frac{3}{5} \cdot \varphi\right)^n - \sum_{n=1}^{\infty} \left(\frac{3}{5} \cdot (1-\varphi)\right)^n \right] \\
 &= \frac{1}{\sqrt{5}} \left[\frac{1}{1 - \frac{3}{5}\varphi} - \frac{1}{1 - \frac{3}{5}(1-\varphi)} \right] \quad (\text{by using the formula for geometric progression}) \\
 &= \frac{1}{\sqrt{5}} \left[\frac{1}{1 - \frac{3}{5}\varphi} - \frac{1}{\frac{2}{5} + \frac{3}{5}\varphi} \right] \\
 &= \frac{1}{\sqrt{5}} \left[\frac{1}{1 - \frac{3}{5}\left(\frac{1+\sqrt{5}}{2}\right)} - \frac{1}{\frac{2}{5} + \frac{3}{5}\left(\frac{1+\sqrt{5}}{2}\right)} \right] \\
 &= \frac{1}{\sqrt{5}} \left[\frac{10}{7 - 3\sqrt{5}} - \frac{10}{7 + \sqrt{5}} \right] \\
 &= \frac{10}{\sqrt{5}} \left[\frac{7 + 3\sqrt{5}}{4} - \frac{7 - 3\sqrt{5}}{4} \right] \\
 &= 15
 \end{aligned}$$

However, we want to use the definitions whereby $F_0=1$, $F_1=1$ and $F_n=F_{n-2}+F_{n-1}$.

Observe that:

$$\frac{[0]}{1} + \frac{3}{5}[1] + \left(\frac{3}{5}\right)^2 [1] + \left(\frac{3}{5}\right)^3 [2] + \dots = 15 \quad (\text{next we times } \frac{5}{3} \text{ on both sides ...})$$

$$0 + [1] + \left(\frac{3}{5}\right)^1 [1] + \left(\frac{3}{5}\right)^2 [2] + \dots = 15 \times \frac{5}{3} = 25$$

$$0 + \frac{F_0}{1} + \frac{3}{5}F_1 + \left(\frac{3}{5}\right)^2 F_2 + \dots = 25$$

Hence, the answer is 25.