

TI-84 Plus Silver Edition Giveaway Puzzle  
 Solution by Nate Burchell

**Question:** One can create a triangle of consecutive positive integers as follows:

```

1
2  3
4  5  6
7  8  9  10
11 12 13 14 15
16 17 18 19 20 21
...
  
```

**Part 1: Given R (row) and C (column), find the value of N = f(R, C), the number at that location of the array.**

The Rth row will have length R and will terminate with the Rth triangular number,  $\frac{R(R+1)}{2}$ . If we start at  $\frac{R(R+1)}{2}$  and subtract R, we will find ourselves at the previous triangular number  $\frac{R(R-1)}{2}$  and we can find the value at the Cth position of the Rth row by adding C. Therefore, the value of

$$f(R,C) = \frac{R(R-1)}{2} + C, \quad 1 \leq C \leq R$$

**Part 2: Given N, find R and C.**

To find the row R in terms of N, we can note again that the Rth row will terminate with the Rth triangular number. The Rth triangular number is  $\frac{R(R+1)}{2}$ , so by solving  $N = \frac{R(R+1)}{2}$  for R, we will get the row in terms of N (if N is the last number of a row) or some number  $R-1 < \text{row} < R$  for numbers which are not the last number in their row. Therefore, we can round up to find which row we are in:

The ceiling function (or least integer function) of the positive solution to  $N = \frac{R(R+1)}{2}$  gives

$$R = \left\lceil \frac{-1 + \sqrt{1 + 8N}}{2} \right\rceil$$

Finally, the column C is given by subtracting from N the first number in N's Row and adding 1:

$$\begin{aligned} C &= N - f(R,1) + 1 \\ &= N - \left( \frac{R(R-1)}{2} + 1 \right) + 1 \end{aligned}$$

This will be the number of positions into the row, or the number of the column, and using the previous expression we could write C in terms of N (without R) if we had a reason to do so.