TI-84 Plus Silver Edition Giveaway Puzzle
Solution by Nate Burchell

Question: One can create a triangle of consecutive positive integers as follows:

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 1
 2   3
 4   5   6
 7   8   9  10
11  12  13  14  15
16  17  18  19  20  21
...
```

Part 1: Given R (row) and C (column), find the value of N = f(*R*, *C*), the number at that location of the array.

The *R*th row will have length *R* and will terminate with the *R*th triangular number, $\frac{R(R+1)}{2}$. If we start at $\frac{R(R+1)}{2}$ and subtract *R*, we will find ourselves at the previous triangular number $\frac{R(R-1)}{2}$ and we can find the value at the *C*th position of the *R*th row by adding *C*. Therefore, the value of

$$f(R,C) = \frac{R(R-1)}{2} + C, \qquad 1 \le C \le R$$

Part 2: Given N, find R and C.

To find the row R in terms of N, we can note again that the Rth row will terminate with the Rth triangular number. The Rth triangular number is $\frac{R(R+1)}{2}$, so by solving $N = \frac{R(R+1)}{2}$ for R, we will get the row in terms of N (if N is the last number of a row) or some number R-1< *row* < R for numbers which are not the last number in their row. Therefore, we can round up to find which row we are in:

The ceiling function (or least integer function) of the positive solution to $N = \frac{R(R+1)}{2}$ gives

$$R = \left\lceil \frac{-1 + \sqrt{1 + 8N}}{2} \right\rceil$$

Finally, the column C is given by subtracting from N the first number in N's Row and adding 1:

$$\begin{aligned} C &= N - f(R,1) + 1 \\ &= N - \left(\frac{R(R-1)}{2} + 1 \right) + 1 \end{aligned}$$

This will be the number of positions into the row, or the number of the column, and using the previous expression we could write C in terms of N (without R) if we had a reason to do so.